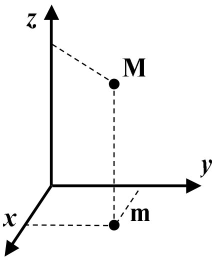
Formulaire d'analyse vectorielle

Calcul vectoriel

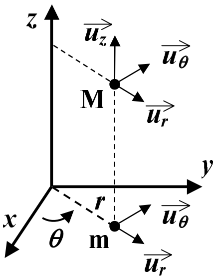
$$
\begin{array}{rcl|rcl}
\overrightarrow{\text{rot}}\ \overrightarrow{\text{grad}}\,V & = & \vec{0} & \overrightarrow{\text{grad}}(V_1V_2) & = & V_1\,\overrightarrow{\text{grad}}\,V_2 + V_2\,\overrightarrow{\text{grad}}\,V_1 \\
\text{div}\,\overrightarrow{\text{rot}}\,\vec{A} & = & 0 & \overrightarrow{\text{rot}}(V\vec{A}) & = & V\,\overrightarrow{\text{rot}}\,\vec{A}\ + \overrightarrow{\text{grad}}\,V \wedge \vec{A} \\
\text{div}\,\overrightarrow{\text{grad}}\,V & = & \Delta V & \text{div}(V\vec{A}) & = & V\,\text{div}\,\vec{A}\ + \overrightarrow{\text{grad}}\,V \cdot \vec{A} \\
\overrightarrow{\text{rot}}\ \overrightarrow{\text{rot}}\,\vec{A} & = & \overrightarrow{\text{grad}}\,\text{div}\,\vec{A} - \Delta\vec{A} & \text{div}(\vec{A}_1 \wedge \vec{A}_2) & = & \vec{A}_2 \cdot \overrightarrow{\text{rot}}\,\vec{A}_1 - \vec{A}_1 \cdot \overrightarrow{\text{rot}}\,\vec{A}_2
\end{array}
$$

Coordonnées cartésiennes



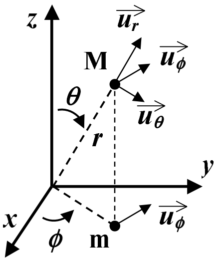
$$
\begin{array}{rcl}
\overrightarrow{\text{grad}}\,V & = & \dfrac{\partial V}{\partial x}\vec{u}_x + \dfrac{\partial V}{\partial y}\vec{u}_y + \dfrac{\partial V}{\partial z}\vec{u}_z \\
\text{div}\,\vec{A} & = & \dfrac{\partial A_x}{\partial x} + \dfrac{\partial A_y}{\partial y} + \dfrac{\partial A_z}{\partial z} \\
\overrightarrow{\text{rot}}\,\vec{A} & = & \left(\dfrac{\partial A_z}{\partial y} - \dfrac{\partial A_y}{\partial z}\right)\vec{u}_x + \left(\dfrac{\partial A_x}{\partial z} - \dfrac{\partial A_z}{\partial x}\right)\vec{u}_y + \left(\dfrac{\partial A_y}{\partial x} - \dfrac{\partial A_x}{\partial y}\right)\vec{u}_z \\
\Delta V & = & \dfrac{\partial^2 V}{\partial x^2} + \dfrac{\partial^2 V}{\partial y^2} + \dfrac{\partial^2 V}{\partial z^2}
\end{array}
$$

Coordonnées cylindriques



$$
\begin{array}{rcl}
\overrightarrow{\text{grad}}\,V & = & \dfrac{\partial V}{\partial r}\vec{u}_r + \dfrac{1}{r}\dfrac{\partial V}{\partial \theta}\vec{u}_\theta + \dfrac{\partial V}{\partial z}\vec{u}_z \\
\text{div}\,\vec{A} & = & \dfrac{1}{r}\dfrac{\partial rA_r}{\partial r} + \dfrac{1}{r}\dfrac{\partial A_\theta}{\partial \theta} + \dfrac{\partial A_z}{\partial z} \\
\overrightarrow{\text{rot}}\,\vec{A} & = & \left(\dfrac{1}{r}\dfrac{\partial A_z}{\partial \theta} - \dfrac{\partial A_\theta}{\partial z}\right)\vec{u}_r + \left(\dfrac{\partial A_r}{\partial z} - \dfrac{\partial A_z}{\partial r}\right)\vec{u}_\theta + \dfrac{1}{r}\left(\dfrac{\partial rA_\theta}{\partial r} - \dfrac{\partial A_r}{\partial \theta}\right)\vec{u}_z \\
\Delta V & = & \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial V}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial^2 V}{\partial \theta^2} + \dfrac{\partial^2 V}{\partial z^2}
\end{array}
$$

Coordonnées sphériques



$$
\begin{array}{rcl}
\overrightarrow{\text{grad}}\,V & = & \dfrac{\partial V}{\partial r}\vec{u}_r + \dfrac{1}{r}\dfrac{\partial V}{\partial \theta}\vec{u}_\theta + \dfrac{1}{r\sin\theta}\dfrac{\partial V}{\partial \phi}\vec{u}_\phi \\
\text{div}\,\vec{A} & = & \dfrac{1}{r^2}\dfrac{\partial r^2A_r}{\partial r} + \dfrac{1}{r\sin\theta}\dfrac{\partial \sin\theta A_\theta}{\partial \theta} + \dfrac{1}{r\sin\theta}\dfrac{\partial A_\phi}{\partial \phi} \\
\overrightarrow{\text{rot}}\,\vec{A} & = & \dfrac{1}{r\sin\theta}\left(\dfrac{\partial \sin\theta A_\phi}{\partial \theta} - \dfrac{\partial A_\theta}{\partial \phi}\right)\vec{u}_r + \ldots \\
 & & \ldots + \dfrac{1}{r}\left(\dfrac{1}{\sin\theta}\dfrac{\partial A_r}{\partial \phi} - \dfrac{\partial rA_\phi}{\partial r}\right)\vec{u}_\theta + \dfrac{1}{r}\left(\dfrac{\partial rA_\theta}{\partial r} - \dfrac{\partial A_r}{\partial \theta}\right)\vec{u}_\phi \\
\Delta V & = & \dfrac{1}{r}\dfrac{\partial^2 rV}{\partial r^2} + \dfrac{1}{r^2\sin\theta}\dfrac{\partial}{\partial \theta}\left(\sin\theta\dfrac{\partial V}{\partial \theta}\right) + \dfrac{1}{r^2\sin^2\theta}\dfrac{\partial^2 V}{\partial \phi^2}
\end{array}
$$

Théorèmes

Théorème d'Ostrogradsky–Green :
S étant une surface fermée, τ le volume intérieur à S,

$$\oint_{(S)} \vec{A} \cdot \mathrm{d}\vec{S} = \int_{(\tau)} (\text{div}\,\vec{A})\mathrm{d}\tau$$

Théorème de Stokes–Ampère :
C étant une courbe fermée bordant une surface S,

$$\oint_{(C)} \vec{A} \cdot \mathrm{d}\vec{l} = \int_{(S)} (\overrightarrow{\text{rot}}\,\vec{A}) \cdot \mathrm{d}\vec{S}$$